

# Synthesis of Wireless Networked Control System Based on Round-trip Delay Online Estimation

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**Abstract**—A control design approach with the integration of online delay estimation is proposed for wireless networked control systems (WNCSs) with unknown round-trip delay, which improves control performance in a practically feasible way. We introduce a delay probability estimation unit to obtain the delay characteristics by estimating the delay when the control system is running. We also present a piecewise approximation control strategy to take advantage of the estimation. Furthermore, the control gain is synthesized with stability guarantee. The conditions to ensure the stochastic stability of the closed-loop system are given, and the effectiveness of the proposed approach is verified numerically.

**Keywords**—Delay characteristics online estimation; Markov jump system; Piecewise approximation control strategy; Wireless network control systems

## I. INTRODUCTION

Wireless network control systems (WNCSs) realize data transmission from sensor to controller and from controller to actuator through wireless network. Compared to its wired counterpart, it has a lot of advantages due to its flexibility in network organizing and building and is easy to deploy, maintain and upgrade [1]. Nowadays, WNCSs have attracted extensive attention in both academia and industry, related application fields include manufacturing executive system (MES) [2], unmanned aerial vehicle control [3], automated warehouse [4], and energy efficient buildings [5].

Although, wireless network brings convenience to control systems, it also brings some imperfections, such as non-zero delay in data transmission and message dropouts and errors and many papers have studied these problems. For example, [6-8] modelled the property of time-varying delay in one or two communication channels as Markov process, and the transition probability matrix is completely known, then the close-loop system can be seen as a Markov jump system, controllers ensure system random stability is designed. Specifically, in [9,10], the authors proposed controller design approaches with uncomplete known transition probability matrix of Markovian delay. In [11,12], communication delay is treated as a random

variable that satisfies a known probability distribution, and the controller is obtained rely on this distribution.

However, the assumption that time delay properties, such as transition probability matrix and probability distribution, are known in advance which most of existing control methods rely on is often not available. Reasons are in the following three aspects. Firstly, transmission network is usually shared with other users, it's difficult for a control system to obtain detailed parameters of the network such as network topology, data transmission speed of each user. Thus, it's difficult to obtain time delay properties through computation and simulation by network parameter [13,14]. Secondly, in WNCSs, control systems can connect to network temporarily, due to the smaller scale of wireless network, after the connect of control system, the topology of network changes relatively greater, result in delay property is greater difference before and after system connect to network [15,16]. So the off-line delay measurement before the control system connect to the network can't correctly reflect the delay property after control system set up and running. Thirdly, after connecting to the network, it takes long time to measure time-delay property online, therefore, the stability of the control system during measurement cannot be guaranteed.

Therefore, delay properties are difficult to predict in advance, which means that many exist controllers above are difficult to be implemented directly in practice. Because of this fundamental difficulty, this paper proposed a control method combined with delay data measured online. A delay estimation module is setup at the controller side to estimate the delay properties by using the online delay data obtained after the control system is connected to the network. At the same time, a triggering condition is designed based on the delay estimation online. When the delay property estimation is not accurate enough, a more conservative controller is used, and the conservatism of controller is gradually reduced with the delay property estimation. In the case of unknown delay properties, by using this simultaneously estimating and triggering control method, the stochastic stability of control system is guaranteed, and the conservativeness of the controller is gradually reduced.

The paper is organized as follows: In Section II, we provide some definitions and formulation the WMCS problem that will

This work is supported by the National Key Research and Development Program of China (No. 2018AAA0100801) and the National Natural Science Foundation of China under Grant 61673350.

be considered in this paper. In Section III, a controller with online estimation of delay properties is proposed. Section IV gives the sufficient conditions for stochastic stability of closed-loop systems and corresponding control gain design method. Section V illustrates the effectiveness of the method through numerical simulation. Section VI concludes this paper.

## II. PROBLEM DESCRIPTION

The structure of the wireless network control system discuss in this paper is shown in Fig. 1. The transmission network used by control system is shared with other users, and the number of nodes and users in the wireless network is relatively small.

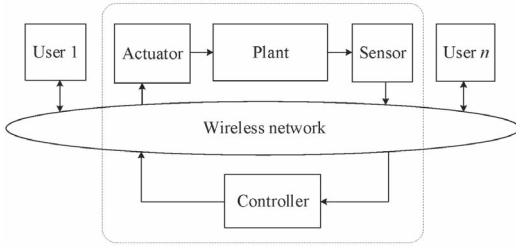


Fig. 1. Framework of wireless networked control system

Consider the linear discrete-time system

$$x(k+1) = Ax(k) + Bu(k) \quad (1)$$

where  $x \in R^n$  is the state of system,  $u \in R^m$  is control input,  $A \in R^{n \times n}$  and  $B \in R^{n \times m}$  are system matrices.

In this system, sensor, controller and actuator are time synchronized. Sensor send measured data to controller through wireless network, controller receives data packets and then updates control action and sends to actuator. The latest packet is selected at the actuator end according to the timestamp of the packet and implements on the controlled plant at execution time.  $\tau_k$  represents the difference between the execution time  $k$  of the actuator and the timestamp corresponding to the execution packet, which is called the round-trip delay at  $k$ -th step.  $\tau_k$  is related to the delay caused by the network, packet loss and so on. Similar to literature [7], we proposed two assumptions for round-trip delay.

**Assumption 1:** There is a upper bound  $M$  for round-trip delay  $\tau_k$ , that is,  $\tau_k \in \mathbf{M} := \{0, 1, 2, \dots, M\}$

**Assumption 2:** The round-trip delay  $\tau_k$  satisfies Markov process with transition probability matrix  $\Pi$ , whose transition probability is

$$\Pr(\tau_{k+1} = j | \tau_k = i) = \begin{cases} \pi_{ij} & j \leq i+1 \\ 0 & j > i+1 \end{cases} \quad (2)$$

where  $\pi_{ij} > 0$ ,  $\forall i, j \in \mathbf{M}$ , and  $\sum_{j=0}^M \pi_{ij} = 1$ .

Since it is difficult for the control system to obtain the delay properties after connecting to the network, we assume that  $\Pi$  is unknown.

Under the assumption above, if we ignore the delay properties, the control system design would be conservative. The goal of this paper is to design a controller to make system (1) to be stochastically stable when the transition probability matrix is unknown.

For subsequent design and analysis, the following definitions are required.

**Definition 1:** The system is stochastically mean square stable, if for any given initial state  $x_0$  and initial delay  $\tau_0$ ,

$$\lim_{k \rightarrow \infty} E(x^T(k)x(k) | x_0, \tau_0) = 0 \quad (3)$$

holds, where  $E(X)$  is the expectation of the random variable  $X$ .

## III. DESIGN METHOD BASED ON ON-LINE MEASUREMENT OF DELAY INFORMATION

For a control system connected to network,  $\Pi$  is unknown after the connection, therefore, system (1) can only be stabilized by a conservative controller, but as time proceeds,  $\Pi$  can be estimated through measuring  $\tau_k$  online. This makes it possible to reduce the conservatism of the controller. At the same time, making full use of delay information is helpful to improve the control performance.

The block diagram of the control system designed in this paper is shown in Fig. 2. The operation flow is described as follows: the sensor sends the sampling timestamp, the round-trip delay data, and the state measurement to the controller together, the Delay Probability Estimation Unit (DPEU) uses the delay data obtained online to obtain the estimation interval of the transfer probability matrix, and sends it to the Control-signal Calculate Unit (CCU), and the resulting control signal is sent to the actuator over the network. The actuator selects the appropriate control signal for the controlled plant through the timestamp and sends the delay data to the sensor.

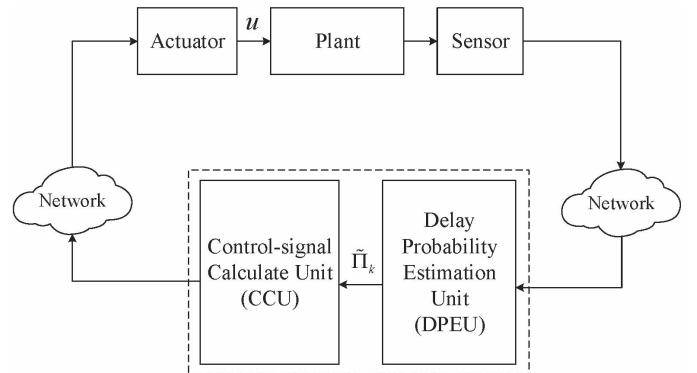


Fig. 2. Control scheme based on on-line delay measurement

Next, we present the detailed description of each unit in the control scheme.

### A. Improved Jeffery interval estimation method

For DPEU, the input is the round-trip delay  $\tau_k$ , and its output is an estimated interval that contains the true value of the probability, denoted by  $\tilde{\Pi}_k$ . In order to achieve this aim, it is necessary for the actuator to send the round-trip delay corresponding to the actuation packet to the sensor, and then

send the round-trip delay to the controller together with the sampling timestamp and the state. For example, the round-trip delay of the  $(k-1)$ -th step,  $\tau_{k-1}$ , will be included in the  $k$ -step packet. The DPEU selects the corresponding data according to the timestamp to estimate.

In traditional Jeffery interval estimation method, with the round trip delay data, take the prior distribution of the estimated value  $\hat{\pi}_{ij,k}$  of  $\pi_{ij}$  as the  $\beta$  distribution with parameter  $a, b$ , i.e.  $\beta(a,b)$ . Estimation interval is  $[\hat{\pi}_{ij,k}, \hat{\pi}_{ij,k}]$ , where  $\hat{\pi}_{ij,k}$  and  $\hat{\pi}_{ij,k}$  are shown in (4)

$$\begin{aligned}\hat{\pi}_{ij,k} &= \beta'(\frac{\alpha}{2}, X_{ij,k}, N_{i,k} - X_{ij,k} + a) \\ \hat{\pi}_{ij,k} &= \beta'(1 - \frac{\alpha}{2}, X_{ij,k}, N_{i,k} - X_{ij,k} + b)\end{aligned}\quad (4)$$

where  $\beta'(h,d,e)$  represents the  $h$  quantile of the  $\beta$  distribution satisfying the parameter  $d, e$ ,  $1-\alpha$  is the confidence level.  $X_{ij,k}$  is the number of delay samples with the delay of  $j$  at the current time when the delay of previous time is  $i$ .  $N_{i,k}$  is total number of samples with delay of  $i$  at the previous time.

If we use Jeffery interval directly, the delay data comes from online measurement, and only one delay data can be obtained per step of control, therefore at the beginning of estimation, there are fewer delay samples, which may lead to the confidence of the estimation interval less than the set value. So this paper improves the Jeffery interval estimation method.

In order to avoid this problem, we develop a learning scheme. When the number of samples is not enough, we makes the estimated interval is close to  $[0,1]$  which is conservative, and when the sample size increases, the trust degree of  $[\hat{\pi}_{ij,k}, \hat{\pi}_{ij,k}]$  is gradually increased. The learning process can be described as follows, where  $\sigma_1$  is a parameter greater than 1.

$$\begin{aligned}\bar{\pi}_{ij,k+1} &= \sigma_1^{-N_{i,k}} + (1 - \sigma_1^{-N_{i,k}}) \hat{\pi}_{ij,k+1} \\ \underline{\pi}_{ij,k+1} &= (1 - \sigma_1^{-N_{i,k}}) \hat{\pi}_{ij,k+1}\end{aligned}\quad (5)$$

The interval of improved Jeffery estimation is  $[\bar{\pi}_{ij,k}, \underline{\pi}_{ij,k}]$ .

We introduce a parameter  $\omega$  to describe the degree of convergence of the estimate,  $\omega$  will be used in controller designing in the Section IV.

$$\omega = \begin{cases} 1 & \max_{\forall i,j \in \mathbf{M}} (\bar{\pi}_{ij,k} - \underline{\pi}_{ij,k}) < \varepsilon \\ 0 & \text{Otherwise} \end{cases}$$

where  $\varepsilon$  is the given threshold. If the width of estimation interval is smaller than threshold, it indicates the estimation is close enough to the true value.

### B. Switching control strategy

For CCU and actuator, a switching control strategy is proposed in this paper, which can reduce the conservatism by using the delay estimation interval obtained online on the premise of ensuring stability of the system.

The controller sets a switching time judgment condition and selects appropriate switching time. The system uses a newer delay transfer probability matrix estimation interval to update control gain at the switching time, and uses the same stabilization control gain in between the two switching times.

The selection of switching time is related to the states of the system. If the two adjacent switching states are guaranteed to meet the decreasing relationship, the control gain will be updated to ensure the stability of the resulting switching system. When the system state is far from the equilibrium point, the switching will not occur, which ensures that the system state tends to the equilibrium point in general. A detailed proof of the stability will be given in Section IV. Therefore, we design the switching condition as (6a) in CCU.

$$z^T(k)z(k) \leq c^{-1}z^T(r_k^-)z(r_k^-) \quad c > 1 \quad (6a)$$

$$k - r_k^- \geq L \quad L \geq M \quad (6b)$$

where  $z^T(k) = (x^T(k), x^T(k-1), \dots, x^T(k-M))$ ,  $r_k^-$  is the latest switching time before time  $k$ , and  $L$  and  $c$  are adjustable parameters. The greater  $L$  and  $c$  are, the greater the interval between two switches. By choosing appropriate  $L$  and  $c$ , we can minimize the computation burden of the controller while the system performance can be guaranteed.

Let the switching times be  $R := \{r_1, r_2, \dots\}$ ,  $r_i$  represents the  $i$ th switching time. In order to ensure that the control signal applied between adjacent switching times comes from the same delay transition probability matrix estimation, and combined with (6b), let the corresponding switching time of  $r_i$  be  $r_i + L$ , and we designed the following control signal transmission and actuation way.

When  $k \in R$ , we update the switching time, and utilize the latest delay transfer probability matrix to estimate interval  $\tilde{\Pi}_k$  to update the control gain. Otherwise, we do not update. Denote the control signals obtained by updated control gain and the previous control gain as  $U_{cur}, U_{old}$ ,

When  $k \in R$

$$\begin{cases} s_k = k + L \\ U_{cur} = K(\tilde{\Pi}_k)x(k), \quad U_{old} = K(\tilde{\Pi}_{r_k^-})x(k) \end{cases} \quad (7)$$

When  $k \notin R$

$$\begin{cases} s_k = r_k^- + L \\ U_{cur} = K(\tilde{\Pi}_{r_k^-})x(k), \quad U_{old} = K(\tilde{\Pi}_{r_k^-})x(k) \end{cases} \quad (8)$$

For the actuator side, at time  $k$ , the actuator takes the latest packet with a timestamp of  $k - \tau_k$ , and reads the switching time in the data packet as  $s_{k-\tau_k}$ . If the current time is not less than the switching time, take  $U_{cur}$  as control signal, otherwise, take  $U_{old}$ , i.e.,

$$u(k) = \begin{cases} K(\tilde{\Pi}_{s_{k-\tau_k}-L})x(k-\tau_k) & k \geq s_{k-\tau_k} \\ K(\tilde{\Pi}_{r_{k-\tau_k}-L})x(k-\tau_k) & k < s_{k-\tau_k} \end{cases} \quad (9)$$

With switching mechanism above, and together with (6b), (7), (8), (9), we can ensure that the controller applied in between two switching times is computed by the same delay transfer probability matrix, as shown in (10).

$$u(k) = K(\tilde{\Pi}_{r_{i-1}})x(k-\tau_k) \quad r_{i-1} + L \leq k < r_i + L \quad (10)$$

where  $r_{i-1} + L, r_i + L$  are the adjacent switching time near to  $k$ .

The control method combined with online delay measurement is summarized as Algorithm 1.

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**Algorithm 1:** Control scheme based on round-trip delay online estimation

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Initialization: Set switching time  $r$  and switching time  $s$  as 0. Send the data packet from the sensor to the controller, which includes sampling timestamp, the latest round-trip delay and  $M$  state vectors.

- 1 The round-trip delay information is sent to the DPEU, and send the state to the CCU.
  - 2 DPEU updates the transition probability matrix according to (4) and (5).
  - 3 The CCU determines whether the received states satisfy the switching condition (6a), (6b) and updates the data packet according to (7), (8) and sends it to the actuator.
  - 4 The actuator selects the latest packet at current sampling time  $k$  and selects the control gain according to (9). At the same time, the latest round-trip delay is sent to the sensor.
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#### IV. STABILITY ANALYSIS AND CONTROLLER DESIGN

In this section, sufficient conditions for the stability of the closed-loop system are given for the controller design method in Section III.

From system (1) and (10), we obtain the closed-loop system as

$$x(k+1) = Ax(k) + BK(\tilde{\Pi}_{r_{i-1}})x(k-\tau_k) \quad r_{i-1} + L \leq k < r_i + L \quad (11)$$

Because the round-trip delay satisfies the Markov process, we can rewrite (11) into the form of Markov jump system

$$z(k+1) = \Phi_{\tau_k}(\tilde{\Pi}_{r_{i-1}})z(k) \quad r_{i-1} + L \leq k < r_i + L \quad (12)$$

where  $\Phi_{\tau_k}(\tilde{\Pi}_{r_{i-1}})$  is shown in (13), and  $I$  is a unit matrix with suitable dimension.  $\Phi_{\tau_k}(\tilde{\Pi}_{r_{i-1}})$  is related to the value of  $\tau_k$ , and it has different representations for different delays, when

delay is  $v$ , then  $BK(\tilde{\Pi}_{r_{i-1}})$  lies in the  $v+1$  column of the first row.

$$\Phi_{\tau_k}(\tilde{\Pi}_{r_{i-1}}) = \begin{bmatrix} A & 0 & \cdots & 0 & BK(\tilde{\Pi}_{r_{i-1}}) & 0 & \cdots & 0 & 0 \\ I & & & & & & & & 0 \\ & I & & & & & & & 0 \\ & & \ddots & & & & & & \vdots \\ & & & I & & & & & 0 \\ & & & & I & & & & 0 \\ & & & & & I & & & 0 \\ & & & & & & I & \ddots & \vdots \\ & & & & & & & & I & 0 \end{bmatrix} \quad (13)$$

The stochastic stability conditions of the closed-loop system are given as follows.

**Theorem 1:** The system (1) is stochastically stable under the designed control strategy. If for  $\forall r \in R$ , there is a corresponding set of symmetric positive definite matrices  $\mathbf{P}_r = \{P_{i,r}, i \in \mathbf{M}\}$ ,  $\eta > 1$ , which makes 2( $M+1$ ) LMIs (14a) and (14b) holds,

$$\Phi_i^T((1-\omega)(\eta-1)(1-\underline{\pi}_{ii,r})P_{i,r} + \omega \sum_{j \neq i}^M \bar{\pi}_{ij,r}P_{j,r})\Phi_i \quad (14a)$$

$$+ \Phi_i^T((1-\omega)P_{i,r} + \omega \bar{\pi}_{ii,r}P_{i,r})\Phi_i - P_{i,r} < 0$$

$$(1-\omega)P_i < \eta P_j \quad \forall i, j \in \mathbf{M} \quad (14b)$$

where  $\bar{\pi}_{\bullet,r}, \underline{\pi}_{\bullet,r}$  are the estimate upper and lower bounds of corresponding transition probability by using the delay data,  $\omega$  indicates whether the estimation is convergent.

**Proof:** For time  $k$ , there must be adjacent switching times  $r_a, r_{a+1}$ , that make  $k \in [r_a + L, r_{a+1} + L)$ . To simplify the notation, we denote the interval as  $[s_k^-, s_k^+)$ . According to (10), in the interval  $[s_k^-, s_k^+)$ , the control gain maintains unchanged. Therefore  $\Phi_{\tau_k}(\tilde{\Pi}_{r_{i-1}})$  in (12) is only related to delay, and we denote it as  $\Phi_{\tau_k}$ .

Consider the following Lyapunov function,

$$V(z(k)) = z^T(k)P_{\tau_k, r_a}z(k)$$

where  $P_{\tau_k, r_a}$  is the positive definite symmetric matrix corresponding to the delays at every time  $k$ , which remains unchanged in between two switching times. Then the difference of the Lyapunov function is

$$E(V(k+1) - V(k) | z(k), \tau_k = i) = z^T(k)(\Phi_i^T(\sum_{j=1}^M \pi_{ij}P_{j, r_a})\Phi_i - P_{i, r_a})z(k) \quad (15)$$

According to the nature of the transition probability, we know

$$\pi_{ii} = 1 - \sum_{i \neq j} \pi_{ij} \quad (16)$$

When  $\omega = 0$ , from (14b) and (16), we can rewrite (15) as

$$\begin{aligned} & z^T(k) (\Phi_i^T (\sum_{j=1}^M \pi_{ij} P_{j,r_a}) \Phi_i - P_{i,r_a}) z(k) \\ &= z^T(k) (\Phi_i^T (\sum_{j \neq i}^M \pi_{ij} (P_{j,r_a} - P_{i,r_a})) \Phi_i + \Phi_i^T P_{i,r_a} \Phi_i - P_{i,r_a}) z(k) \\ &\leq z^T(k) (\Phi_i^T ((\eta-1) \sum_{j \neq i}^M \pi_{ij} P_{i,r_a}) \Phi_i + \Phi_i^T P_{i,r_a} \Phi_i - P_{i,r_a}) z(k) \\ &< z^T(k) (\Phi_i^T (\eta-1) (1 - \underline{\pi}_{ii,r}) \Phi_i + \Phi_i^T P_{i,r_a} \Phi_i - P_{i,r_a}) z(k) \end{aligned} \quad (17)$$

(14a) guarantees (17) is less than 0, thus there exists  $\gamma_1$ ,  $0 < \gamma_1 < 1$ , such that (18) holds

$$E(V(k+1) | z(k), \tau_k) \leq \gamma_1 V(k) \quad (18)$$

For the same reason, when  $\omega = 1$ , (18) also holds.

Since the control gain sequence used at any time between the two switching times is constant, recur (17) we can obtain

$$E(V(k) | z(s_k^-), \tau_{s_k^-}) < \gamma_1^{k-s_k^-} z^T(s_k^-) P_{\tau_{s_k^-}, r_a} z(s_k^-) \quad (19)$$

From (19) we can obtain

$$\begin{aligned} & E(z^T(k) z(k) | z(s_k^-), \tau_{s_k^-}) \leq \gamma_1^{k-s_k^-} \lambda_1 z^T(s_k^-) z(s_k^-) \\ & \lambda_1 = \max_{i,j \in \mathbf{M}} (\lambda_{\max}(P_{i,r_a}) / \lambda_{\min}(P_{j,r_a})) \end{aligned} \quad (20)$$

where  $\lambda_{\max}(\bullet)$ ,  $\lambda_{\min}(\bullet)$  indicate the maximum and minimum eigenvalues of the matrix.

Since we keep the control gain obtain at time  $r_a$  constant for the interval between  $r_a$  and the next switch time  $s_k^-$ , we can get (21) which describes the relationship of states in between switch instants,

$$E(z^T(k) z(k) | z(r_a), \tau_{r_a}) \leq \gamma_2^L \lambda_2 z^T(r_a) z(r_a) \quad (21)$$

From (6a), (20) and (21), we obtain

$$E(z^T(k) z(k) | z_0, \tau_0) \leq \gamma_2^L \gamma_1^{k-s_k^-} \lambda_1 \lambda_2 c^{-w} z_0^T z_0 \quad (22)$$

Therefore

$$\lim_{k \rightarrow \infty} E(z^T(k) z(k) | z_0, \tau_0) = \lim_{w \rightarrow \infty} \gamma_2^L \gamma_1^{k-s_k^-} \lambda_1 \lambda_2 c^{-w} z_0^T z_0 = 0 \quad (23)$$

It satisfies the definition of stochastically stable, thus Theorem 1 is proved. ■

**Corollary 1:** The system (1) is stochastically stable under the designed control strategy. If for  $\forall r \in R$ , there is a corresponding set of symmetric positive definite matrices  $\mathbf{P}_r = \{P_{i,r}, i \in \mathbf{M}\}$ , control gain  $K$ , and  $\eta > 1$ , such that LMIs (24) and (25) holds,

$$\begin{bmatrix} -P_{i,r} & T_{i,r}^T \\ T_{i,r} & -\Gamma_r \end{bmatrix} < 0 \quad (24)$$

$$(1-\omega)P_{i,r} < \eta P_{j,r} \quad \forall i, j \in \mathbf{M} \quad (25)$$

where

$$\begin{aligned} T_{i,r}^T &= [((1-\omega)\sqrt{(\eta-1)(1-\underline{\pi}_{ii,r})}/M + \omega\sqrt{\underline{\pi}_{i1,r}})\Phi_i^T, \\ &\quad \dots, ((1-\omega)\sqrt{(\eta-1)(1-\underline{\pi}_{ii,r})}/M + \omega\sqrt{\underline{\pi}_{ii-1,r}})\Phi_i^T, \\ &\quad ((1-\omega)\sqrt{(\eta-1)(1-\underline{\pi}_{ii,r})}/M + \omega\sqrt{\underline{\pi}_{ii+1,r}})\Phi_i^T, \\ &\quad \dots, ((1-\omega) + \omega\sqrt{\underline{\pi}_{ii,r}})\Phi_i^T] \end{aligned}$$

$$\Gamma_r = \text{diag}((1-\omega)P_{i,r}^{-1} + \omega P_{1,r}^{-1}, \dots, (1-\omega)P_{i,r}^{-1} + \omega P_{M,r}^{-1}, P_{i,r}^{-1})$$

This corollary can be proved by applying Schur complement on LMIs in Theorem 1.

In (24) and (25),  $\eta$  is coupled with  $\mathbf{P}_r$  and  $K$  when  $\omega=0$ . The following off-line solution Algorithm 2 is designed to get  $\eta$ . Then  $K$  can be obtained by cone complementarity linearization (CCL) algorithm in [17].

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#### Algorithm 2: Finding solution of $\eta$ off-line

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- 1 Input :  $\underline{\pi}_{ii,0} = 0$ , a given step size  $N$ ,  $\eta=1$ ,  $\omega=0$
  - 2 While (24) has solution, do
  - 3      $\eta = \eta + N$
  - 4 While (24) and (25) has no solution, do
  - 5      $\eta = \eta - N$
  - 6 Output:  $\eta$
- 

## V. NUMERICAL EXAMPLE

Consider linear discrete system (1), and the system matrices are given as,

$$A = \begin{bmatrix} 0.7769 & 0.25 \\ 0.25 & 1.0163 \end{bmatrix} \quad B = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}$$

The eigenvalues of the system are 0.6193 and 1.1738, respectively. The open loop system is an unstable system. The initial states  $x(0)=[5,5]^T$ , and initial delay  $\tau_0=1$ .

For the network delay properties, the upper bound  $M$  of the packet round-trip delay is 4. In the simulation, the delay transition probability matrix is set as  $\Pi$ :

$$\Pi = \begin{bmatrix} 0.1 & 0.9 & 0 & 0 & 0 \\ 0.2 & 0.1 & 0.7 & 0 & 0 \\ 0.1 & 0.2 & 0.2 & 0.5 & 0 \\ 0.15 & 0.15 & 0.2 & 0.2 & 0.3 \\ 0.05 & 0.1 & 0.15 & 0.3 & 0.4 \end{bmatrix}$$

DPEU estimates  $\Pi$  by delay data. In Fig. 3, we plot estimation interval of  $\pi_{23}$  get by improved Jeffrey interval  $[\bar{\pi}_{23,k}, \underline{\pi}_{23,k}]$  (solid line), the estimation parameters are  $\sigma_1=1.01$ ,  $\alpha=0.01$ , and traditional Jeffrey interval  $[\hat{\pi}_{23,k}, \hat{\pi}_{23,k}]$  (dotted line). Fig. 3 shows that traditional Jeffrey interval may not cover 0.7 at beginning of the time evolution, but improved Jeffrey interval contains 0.7 all the time.

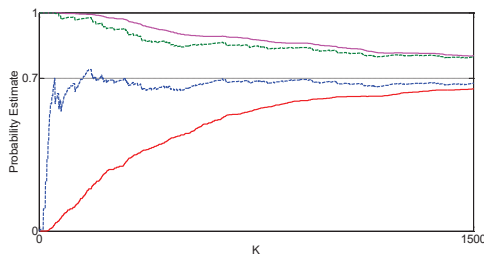


Fig. 3. Estimation trajectories of  $[\bar{\pi}_{23,k}, \underline{\pi}_{23,k}]$  and  $[\hat{\pi}_{23,k}, \hat{\pi}_{23,k}]$

Next, the state trajectories of the closed-loop system under the switching control strategy are shown in Fig. 4, it is shown that the switching control strategy can make the system stable.

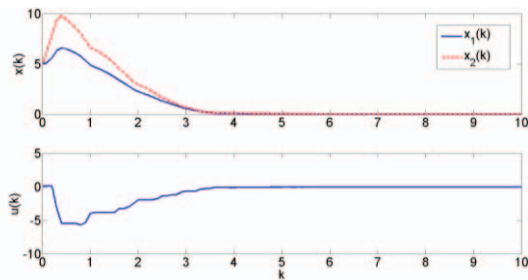


Fig. 4. State and control trajectories under the switching control strategy

## VI. CONCLUSION

This paper proposed a switching control strategy with round-trip delay online measurement data for wireless networked control systems when the delay properties are unknown. A delay probability estimation unit, which gradually reduces the conservatism of the control, is integrated in the control system which employs the latest delay transfer probability matrix to update the control gain during the switching interval. The effectiveness of the control strategy is verified by a numerical example.

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